

**FINALTERM EXAMINATION 2009**  
**(Session - 1)**  
**Calculus & Analytical Geometry-I**

**Question No: 1 ( Marks: 1 ) - Please choose one**

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If  $f$  is a twice differentiable function at a stationary point  $x_0$  and  $f''(x_0) > 0$   
then  $f$  has relative ..... At  $x_0$

- ▶ Minima
- ▶ Maxima
- ▶ None of these

**Question No: 2 ( Marks: 1 ) - Please choose one**

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If  $f$  is a twice differentiable function at a stationary point  $x_0$  and  $f''(x_0) < 0$   
then  $f$  has relative ..... At  $x_0$

- ▶ Minima
- ▶ Maxima
- ▶ None of these

**Question No: 3 ( Marks: 1 ) - Please choose one**

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$$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \text{-----}$$

- ▶ 2
- ▶ 4

- ▶ 1
- ▶  $\infty$

**Question No: 4 ( Marks: 1 ) - Please choose one**

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$$\lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \text{-----}$$

- ▶ 1
- ▶ 0
- ▶ e
- ▶ None of these

**Question No: 5 ( Marks: 1 ) - Please choose one**

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$$\frac{d(\tan x)}{dx} =$$

- ▶  $\sec x$
- ▶  $\sec^2 x$
- ▶  $\operatorname{cosec} x$
- ▶  $\operatorname{cosec}^2 x$

**Question No: 6 ( Marks: 1 ) - Please choose one**

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$$\text{If } xy = 4 \text{ then } \frac{dy}{dx} =$$

- ▶ 0
- ▶  $\frac{-1}{x^2}$
- ▶  $\frac{4}{x^2}$
- ▶  $\frac{-4}{x^2}$

**Question No: 7 ( Marks: 1 ) - Please choose one**

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Consider a function  $h(x)$  and a constant  $c$  then

$$\frac{d}{dx}((c) \{h(x)\}) = \underline{\hspace{2cm}}$$

- ▶ 0
- ▶  $\frac{d}{dx}(h(x))$
- ▶  $\frac{d}{dx}(h(cx))$
- ▶  $c \frac{d}{dx}(h(x))$

**Question No: 8 ( Marks: 1 ) - Please choose one**

Suppose that  $f$  and  $g$  are differentiable functions of  $x$  then

$$\frac{d}{dx}[f][g] =$$

- ▶  $\frac{[f'] [g] - [f] [g']}{g^2}$
- ▶  $[f'] [g']$
- ▶  $[f'] [g] + [f] [g']$
- ▶  $[f'] [g] - [f] [g']$

**Question No: 9 ( Marks: 1 ) - Please choose one**

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

The power rule, \_\_\_\_\_ holds if n is \_\_\_\_\_

- ▶ An integer
- ▶ A rational number
- ▶ An irrational number
- ▶ All of the above

**Question No: 10 ( Marks: 1 ) - Please choose one**

Let a function  $f$  be defined on an interval, and let  $x_1$  and  $x_2$  denotes two distinct points in that interval. If  $f(x_1) = f(x_2)$  for all points  $x_1$  and  $x_2$  then

which of the following statement is correct?

- ▶  $f$  is a decreasing function
- ▶  $f$  is an increasing function
- ▶  $f$  is a constant function

**Question No: 11 ( Marks: 1 ) - Please choose one**

If  $f''(x) < 0$  on an open interval (a,b) then which of the following statement is correct?

- ▶  $f$  is concave up on (a, b).
- ▶  $f$  is concave down on (a, b)
- ▶  $f$  is linear on (a, b).

**Question No: 12 ( Marks: 1 ) - Please choose one**

$$\sum_{k=1}^n f(x_k^*) \Delta x_k$$

What does 'n' represent in Riemann Sum ?

- ▶ No. of Circles
- ▶ No. of Rectangles
- ▶ No. of Loops
- ▶ No. of Squares

**Question No: 13 ( Marks: 1 ) - Please choose one**

$$\lim_{x \rightarrow -\infty} f(x) = +\infty \text{ and } \lim_{x \rightarrow +\infty} f(x) = +\infty$$

If  $f$  is continuous function such that  $(-\infty, +\infty)$  then  $f$  has \_\_\_\_\_ on

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

**Question No: 14 ( Marks: 1 ) - Please choose one**

$$\int_2^t \frac{x^2}{2} dx$$

The expression \_\_\_\_\_, represents a function of :

- ▶  $t$
- ▶  $x$
- ▶ 2
- ▶ Both  $t$  and  $x$

**Question No: 15 ( Marks: 1 ) - Please choose one**

$$\int cf(x)dx = \underline{\hspace{2cm}}$$

if c is a constant

▶ 0

▶ c

▶  $\int f(cx)dx$

▶

▶  $c \int f(x)dx$

▶

**Question No: 16 ( Marks: 1 ) - Please choose one**

Sigma notation is represented by which of the following Greek letter?

▶  $\chi$

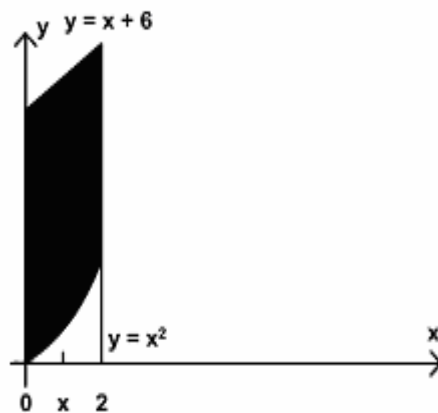
▶  $\eta$

▶  $\Sigma$

▶  $\psi$

**Question No: 17 ( Marks: 1 ) - Please choose one**

In the following figure, the area enclosed is bounded below by :



▶  $y = x + 6$

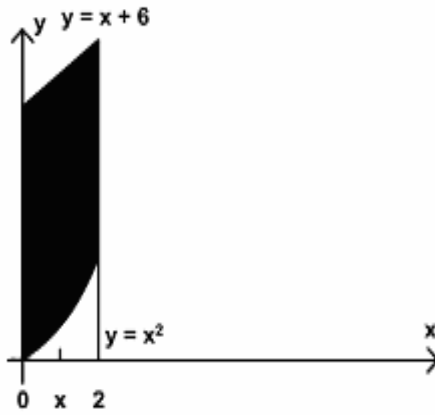
▶  $y = x^2$

▶  $x = 2$

►  $x = 0$

**Question No: 18 ( Marks: 1 ) - Please choose one**

In the following figure, the area bounded on the sides by the lines are :



►  $x = 0$

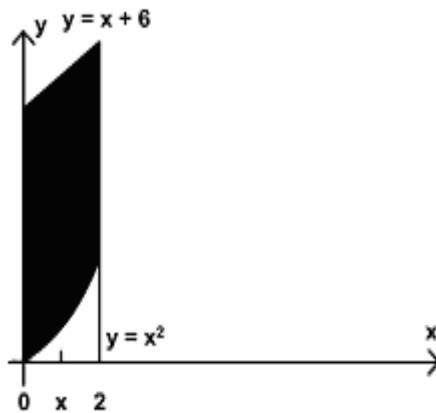
►  $x = 2$

►  $x = 0$  and  $x = 2$

►  $x = 6$

**Question No: 19 ( Marks: 1 ) - Please choose one**

What is the area of the region in the following figure?



► 
$$A = \int_0^2 [(x+6) - (x^2)] dx$$

► 
$$A = \int_x^2 [(x+6) - (x^2)] dx$$

►

$$A = \int_0^2 [(x+6) + (x^2)] dx$$



$$A = \int_0^x [(x+6) - (x^2)] dx$$



**Question No: 20 ( Marks: 1 ) - Please choose one**

Which of the following is approximate area under the curve  $y = f(x) = 3x + 1$  over the interval  $[2, 4]$ , evaluated by using the formula

$$Area = f(x_1^*) \Delta x + f(x_2^*) \Delta x$$

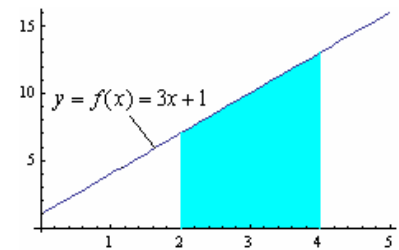
If the interval  $[2, 4]$  is divided into two sub-intervals of equal

length and  $x_1^*$  and  $x_2^*$  are left endpoint of each sub-interval.

► 17

► 20

► 23



**Question No: 21 ( Marks: 1 ) - Please choose one**

Which of the following is approximate area under the curve  $y = f(x) = 2x + 3$  over the interval  $[0, 2]$ , evaluated by using the formula

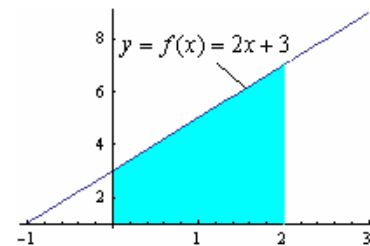
$$Area = f(x_1^*) \Delta x + f(x_2^*) \Delta x$$

If the interval  $[0, 2]$  is divided into two sub-intervals of equal length and  $x_1^*$  and  $x_2^*$  are right endpoint of each sub-interval.

► 8

► 10

► 12



**Question No: 22 ( Marks: 1 ) - Please choose one**

If  $x > 0$  then  $\frac{d}{dx}[\ln x] =$  \_\_\_\_\_

►  $1$

►  $x$

►  $\frac{1}{x}$

►  $\ln \frac{1}{x}$

**Question No: 23 ( Marks: 1 ) - Please choose one**

Suppose  $f$  and  $g$  are integrable functions on  $[a,b]$  and  $c$  is a constant, then

$\int_a^b c [f(x) + g(x)] dx =$  \_\_\_\_\_

►  $\int_a^b f(cx)dx + \int_a^b g(cx)dx$

►

$\int_a^b f(x) dx + \int_a^b g(x) dx$

►

►  $c \int_a^b f(x)dx + c \int_a^b g(x)dx$

►  $0$

**Question No: 24 ( Marks: 1 ) - Please choose one**

If the function  $f$  is continuous on  $[a,b]$  and if  $f(x) \geq 0$  for all  $x$  in  $[a,b]$ , then which of the following gives area under the curve  $y = f(x)$  over the interval  $[a,b]$ ?

$\lim_{n \rightarrow \infty} \sum_{k=1}^n [x_k][f(x_k)]$  where  $n$  is number of subdivisions of  $[a,b]$

►

►  $\int_a^b f(x) dx$

►

►  $\pi[\text{radius}]^2$

►

► (Width) (Height)



**Question No: 25 ( Marks: 1 ) - Please choose one**

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Let region R in the first quadrant enclosed between  $y = 3x$  and  $y = 2x^2$  is revolved about the x-axis .Which of the following equation gives the volume of a solid by cylindrical shells?

▶ 
$$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$$

▶ 
$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$$

▶ 
$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

▶ 
$$V = \int_{-1}^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

**Question No: 26 ( Marks: 1 ) - Please choose one**

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Let f is a smooth function on  $[a, b]$ . What will be the arc length L of the curve  $y = f(x)$  from  $x = a$  to  $x = b$ ?

▶ 
$$L = \int_b^a \sqrt{1 + [f'(x)]} dy$$

▶ 
$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$L = \int_0^a \sqrt{1 + [f'(x)]} dy$$



$$L = \int_a^b \sqrt{1 + [f'(x)]} dx$$



**Question No: 27 ( Marks: 1 ) - Please choose one**

If f is continuous on (a, b] but does not have a limit from the right then the

$$\int_a^b f(x) dx = \lim_{l \rightarrow a} \int_l^b f(x) dx$$

integral defined by

is called :

► Improper

► Proper

► Line

**Question No: 28 ( Marks: 1 ) - Please choose one**

$$\frac{a_{n+1}}{a_n} > 1$$

For a sequence  $\{a_n\}$  if the ratio of successive terms then the sequence is known as:

► Increasing

► Decreasing

► Nondecreasing

► Nonincreasing

**Question No: 29 ( Marks: 1 ) - Please choose one**

$$\frac{a_{n+1}}{a_n} < 1$$

For a sequence  $\{a_n\}$  if the ratio of successive terms then the sequence is known as:

► Increasing

► Decreasing

► Nondecreasing

► Nonincreasing

**Question No: 30 ( Marks: 1 ) - Please choose one**

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$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx$$

Consider the indefinite integral

Let  $t = x^3 + 2x^2 + x - 3$

Is the following substitution correct?

$$\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 3} dx = \int \frac{1}{t} dt$$

► Yes

► No

**Question No: 31 ( Marks: 1 ) - Please choose one**

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$$\rho = \lim_{k \rightarrow \infty} \frac{u_{k+1}}{u_k}$$

The series  $\sum u_k$  be a series with positive terms and suppose that if  $\rho = 1$ , then which of the following is true?

► Converges

► Diverges

► May converges or diverges

► Gives no information

**Question No: 32 ( Marks: 1 ) - Please choose one**

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The series  $\sum u_k$  be a series with positive terms and suppose that

$\rho = \lim_{k \rightarrow \infty} \sqrt[k]{u_k} = \lim_{k \rightarrow \infty} (u_k)^{\frac{1}{k}}$  if  $\rho = 1$ , then which of the following is true?

► Converges

► Diverges

► May converges or diverges

► Gives no information

**Question No: 33 ( Marks: 1 ) - Please choose one**

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$$\sum_{k=1}^{\infty} |u_k| = |u_1| + |u_2| + |u_3| + \dots + |u_k| + \dots$$

If the series \_\_\_\_\_ converges , then which of

$$\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \dots + u_k + \dots$$

the following is true for \_\_\_\_\_ ?

- ▶ Converges
- ▶ Diverges
- ▶ Gives no information

**Question No: 34 ( Marks: 1 ) - Please choose one**

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$$\rho = \lim_{k \rightarrow \infty} \frac{|u_{k+1}|}{|u_k|}$$

Let  $\sum u_k$  be a series with nonzero terms and suppose that if  $\rho = +\infty$  , then which of the following is true?

- ▶ Then the series  $\sum u_k$  diverges
- ▶ The series  $\sum u_k$  converges absolutely and therefore converges
- ▶ May converges or diverges
- ▶ Gives no information

**Question No: 35 ( Marks: 1 ) - Please choose one**

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$$\int_{-1}^1 (x-1) dx = \underline{\hspace{2cm}}$$

- ▶ -2
- ▶ 0
- ▶ 2
- ▶ 4

**Question No: 36 ( Marks: 1 ) - Please choose one**

---

How many critical points exist for a function  $f$  if

$$f'(x) = (x-3)(x-2)$$

- ▶ Zero
- ▶ One
- ▶ Two
- ▶ Four

**Question No: 37 ( Marks: 1 ) - Please choose one**

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$$\log_b ac = \underline{\hspace{2cm}}$$

- ▶  $\log_b a + \log_b c$
- ▶  $\log_b a - \log_b c$
- ▶  $\frac{\log_b a}{\log_b c}$
- ▶  $(\log_b a)(\log_b c)$
- ▶

**Question No: 38 ( Marks: 1 ) - Please choose one**

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$$\log_b a^r = \underline{\hspace{2cm}}$$

- ▶  $a \log_b r$
- ▶  $r \log_b a$
- ▶  $\frac{\log_b a}{\log_b r}$
- ▶
- ▶  $\log_b a + \log_b r$

**Question No: 39 ( Marks: 1 ) - Please choose one**

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$$y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} ; 0 \leq x \leq 2$$

Let \_\_\_\_\_ then which of the following is the length of the curve?

$$L = \int_0^2 \sqrt{\left[ \frac{d}{dx} \left( \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int \sqrt{1 + \left[ \frac{d}{dx} \left( \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left[ \frac{d}{dx} \left( \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



$$L = \int_0^2 \sqrt{1 + \left[ \frac{d}{dx} \left( \frac{2\sqrt{2}}{3} x^{\frac{3}{2}} \right) \right]^2} dx$$



**Question No: 40 ( Marks: 1 ) - Please choose one**

Which of the following are *first two* terms for the Taylor series of  $f(x) = e^{-x}$  at  $x = 0$ ?

$$1 + (1)(x - 0)$$



$$1 + (-1)(x + 0)$$



$$1 + (-1)(x - 0)$$



$$(-1)(x - 0)$$



**Question No: 41 ( Marks: 2 )**

$$\int_2^3 (1 - x) dx$$

Evaluate the integral

$$\begin{aligned}
 & \int_2^3 (1-x) dx \\
 &= \left[ x - x^2 \frac{1}{2} \right]_2^3 \\
 &= \frac{1}{2} [2x - x^2]_2^3 \\
 &= \frac{1}{2} (2(3-2) - (3-2)^2) \\
 &= \frac{1}{2} (2-1) \\
 &= \frac{1}{2}
 \end{aligned}$$

**Question No: 42 ( Marks: 2 )**

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$$\int_2^{+\infty} \frac{dx}{x^2}$$

Evaluate the improper integral

**Question No: 43 ( Marks: 2 )**

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A function  $f(x) = x^2 - 4x - 9$  has critical point 2 in an interval  $[0, 5]$ . Find the maximum value of the function and point having this value.

**Question No: 44 ( Marks: 3 )**

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$$\int \frac{5 - 6 \sin^2 x}{\sin^2 x} dx$$

Evaluate:

$$\int \frac{5 - 6 \sin^2 x}{\sin^2 x} dx$$

**Question No: 45 ( Marks: 3 )**

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Find the area of the region bounded by the curve  $y = x^2$ ,  $x > 0$ , and bounded on the sides by the lines  $y = 1$  and  $y = 4$

$$y = x^2, \quad x > 0$$

So we have

$$\begin{aligned}
 A &= \int_1^4 x^2 dx \\
 &= \left. \frac{x^3}{3} \right|_1^4 \\
 &= \frac{1}{3} (4-1)^3 \\
 &= \frac{1}{3} (3)^3 \\
 &= 9
 \end{aligned}$$

---

**Question No: 46 ( Marks: 3 )**

Determine whether the following sequence converges or diverges. If it converges, find the limit.

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$$\lim_{n \rightarrow \infty} \frac{5n^2 - 1}{20n + 7n^2}$$

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**Question No: 47 ( Marks: 5 )**

Use the Alternating series Test to determine whether the given series converges

$$\sum_1^{\infty} \frac{(-1)^{n-1} \cdot n!}{2^n}$$

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**Question No: 48 ( Marks: 5 )**

Evaluate the integral

$$\int_{\frac{\pi}{2}}^0 \frac{1 + \cos 2t}{2} dt$$

Solution



$$\int_{\frac{\pi}{2}}^0 \frac{1 + \cos 2t}{2} dt$$

$$u = 2t$$

$$\frac{du}{dt} = 2$$

$$du = 2dt$$

so

$$= \frac{1}{4} \int_{\frac{\pi}{2}}^0 1 + \cos u du$$

$$= \frac{1}{4} [u + \sin u]_{\frac{\pi}{2}}^0$$

$$= \frac{1}{4} [2t + \sin 2t]_{\frac{\pi}{2}}^0$$

$$= \frac{1}{4} (2 \cdot \frac{\pi}{2} + \sin 2 \cdot \frac{\pi}{2})$$

$$= \frac{1}{4} (\pi + \sin \pi)$$

$$= \frac{1}{4} (\pi + 0)$$

$$= \frac{\pi}{4}$$

#### Question No: 49 ( Marks: 5 )

Evaluate the sums

$$\sum_{k=1}^5 k(3k+5)$$

$$= 1(3+5) + 2(6+5) + 3(9+5) + 4(12+5) + 5(15+5)$$

$$= 8 + 22 + 3(45) + 4(60) + 5(75)$$

$$= 8 + 22 + 135 + 240 + 375$$

$$= 780$$

#### Question No: 50 ( Marks: 10 )

Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x – axis.

$$y = 1 + x^3, \quad x = 1, x = 2, y = 0$$

$$\text{from } V = \int_a^b \pi [f(x)]^2 dx$$

$$V = \int_1^2 \pi [1 + x^3]^2 dx$$

$$V = \int_1^2 \pi [1 + x^5 + 2x^3] dx$$

$$V = \pi \int_1^2 (1 + x^5 + 2x^3) dx$$

$$V = \pi \left( x + \frac{1}{6} x^6 + \frac{1}{2} x^4 \right) \Big|_1^2$$

$$V = \pi \left( (2 - 1) + \frac{1}{6} (2 - 1)^6 + \frac{1}{2} (2 - 1)^4 \right)$$

$$V = \pi \left\{ ((2 - 1) + \frac{1}{6} (2 - 1)^6 + \frac{1}{2} (2 - 1)^4) \right\}$$

$$V = \pi \left( (1) + \frac{1}{6} + \frac{1}{2} \right)$$

$$V = \frac{\pi(6 + 1 + 3)}{6}$$

$$V = \frac{\pi(10)}{6} = \pi \frac{5}{3}$$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at [gulshanvu@yahoo.com](mailto:gulshanvu@yahoo.com), with reference to the concerned paper's number.